

Verifying Software Emulation of an Unsupported Hardware Instruction [DRAFT]

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Abstract

Some processors, especially embedded ones, do not implement all instructions in hardware. Instead, if the processor encounters an unimplemented instruction, an unsupported-instruction exception is raised, and an exception handler is run which implements the missing instruction in software. Getting such a system to work correctly is tricky: The exception handler code must not destroy any state of the user program and must use the control and status registers (CSRs) of the processor correctly. Moreover, parts of the handler are typically implemented in assembly, while other parts are implemented in a language like C, and one must make sure that when jumping from the user program into the handler assembly, from the handler assembly into C, back to assembly and finally back to the user program, all the assumptions made by the different pieces of code, hardware, and the compiler are satisfied.

Despite all these tricky details, there is a concise and intuitive way of stating the correctness of such a system: User programs running on a system where some instructions are implemented in software behave the same as if they were running on a system where all instructions are implemented in hardware.

We formalize and prove such a statement in the Coq proof assistant, for the case of a simple exception handler implementing the multiplication instruction on a RISC-V processor.

2012 ACM Subject Classification [Replace ccsdesc macro with valid one](#)

Keywords and phrases Software verification, Software-hardware boundary, Coq

1 Introduction

Assembly language is frequently regarded as the lowest level of software abstraction in software verification endeavors. However, the ISA (Instruction Set Architecture) semantics typically employed for software verification present an abstraction of the bare-metal ISA specifications, omitting machine-level aspects of the ISA, like the configuration registers that control the intricate interplay between the hardware's intrinsic capabilities and the meticulously crafted firmware (a piece of software) tasked with maintaining machine configurations and implementing high-privilege handlers in charge of emulating unsupported instructions, and managing other forms of low-level exceptions.

For example, in the RISC-V ISA, Control and Status Registers (CSRs) shape the behavior and functionality of the machine. These registers serve as a mechanism for controlling various aspects of the processor's operation, ranging from enabling or disabling specific features to controlling where the machine jumps in case of interrupts and exceptions. These registers and the associated exception handlers exert fundamental control over machine behaviors, so their improper configuration can lead to undefined outcomes.

Control and Status Registers coupled with the handlers introduce an intriguing specification, implementation, and verification challenge: while they are both essential to determining

45 the machine’s behavior, the Control and Status registers are themselves set and manipulated
 46 by software, and the handlers are themselves software.

47 There is a bit of a chicken-and-egg problem: We want to provide a nice and simple ISA
 48 abstraction, but to implement this abstraction and prove it correct, we have to write a trap
 49 handler and want to use a compiler whose proof already relies on this abstraction that we
 50 are supposed to implement, so how can we break the circularity?

51 Let’s acknowledge that one might be tempted to simply augment software verification
 52 efforts with more detailed and faithful ISA specifications. We eschew this approach. The
 53 simplified ISA abstractions commonly employed are far more practical and productive
 54 compared to their cumbersome and heavier bare-metal counterparts, and the intricate details
 55 of configurations and handlers should anyway remain irrelevant to software or compilers
 56 higher up the stack.

57 This paper endeavors to disentangle the problem by focusing on a *simplified-yet-illustrative*
 58 instance: the specification, implementation, and verification of a RISC-V machine with
 59 software-implemented multiply instructions.

60 Through this exploration, we aim to shed light on the interesting challenges posed by
 61 Control and Status Registers and handlers and pave the way for a more coherent understanding
 62 of hardware-software interactions.

63 We will show that for this simple case we can indeed provide (with proofs!) the desired
 64 abstractions, and that, maybe counterintuitively, we can leverage tools that were built on
 65 top of those nice abstractions to provide the said abstractions, without creating a circular
 66 conundrum.

67 Our paper makes the following contributions:

- 68 ■ We propose a pleasantly simple specification for a RISC-V system equipped with a
 69 software trap handler emulating unsupported instructions: User programs running on a
 70 system where some instructions are implemented in software in a trap handler should
 71 behave as if they were running on a system with hardware support for these instructions.
- 72 ■ We implement such a trap handler by combining code in a C-like language with hand-
 73 written assembly code, and prove its correctness, in a mechanized and foundational way,
 74 down to the binary machine code of the handler, combining symbolic-evaluation proofs
 75 at the C-level and assembly level with a compiler-correctness proof.

76 All our code is publicly available at <https://github.com/mit-plv/softmul>.

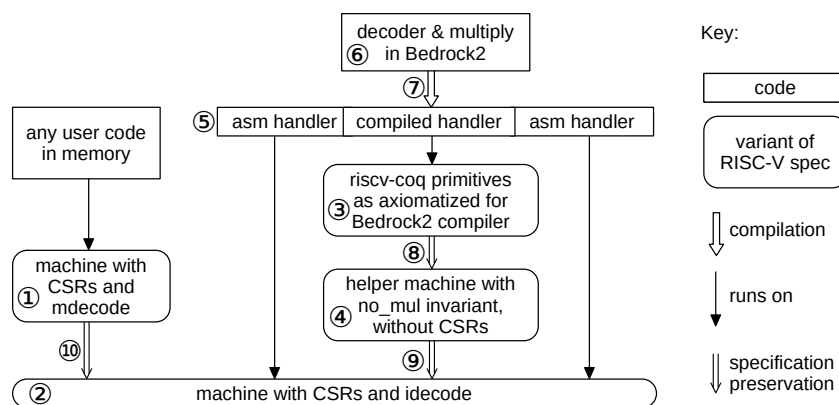
77 2 Overview

78 We want to show that a machine without hardware support for multiplication, but correctly
 79 configured with an exception handler that implements multiplication in software, behaves
 80 like a machine that supports multiplication in hardware. This theorem could then be used to
 81 simplify reasoning about programs running on a machine without hardware multiplication,
 82 because it saves the burden of reasoning about the trap handler and instead makes it as easy
 83 as reasoning about the specification with multiplication in hardware:

```

84 match inst with
  | Mul rd rs1 rs2 =>
    x ← getRegister rs1;
    y ← getRegister rs2;
    setRegister rd (mul x y)
  | ...
end

```



■ **Figure 1** Overview diagram. The circled numbers are referenced in the text and do not stand for any meaningful order.

85 We use the RISC-V instruction set architecture [1, 2], as formalized in riscv-coq [5].
 86 RISC-V splits the instruction set into several extensions, each named with an uppercase letter.
 87 The base instruction set that every processor must support is called I, and multiplication,
 88 division and modulo operations are in a separate extension called M that small embedded
 89 processors may choose not to implement, or to implement in software by catching unsupported-
 90 instruction exceptions.

91 The riscv-coq specification defines a set of roughly a dozen *primitives* such as e.g.
 92 `getRegister`, `setRegister`, `loadByte`, `storeByte`, and then defines the semantics of each RISC-V
 93 instruction in terms of these primitives. As explained in [5], the semantics of each primitive
 94 is deliberately left unspecified in riscv-coq, so that each application that needs a formal
 95 specification of RISC-V can instantiate these primitives in a suitable domain-specific way.

96 Figure 1 presents an overview of our code (boxes ⑤ and ⑥) and specifications (the
 97 remaining boxes). Our theorem uses two instantiations of the riscv-coq specification: One
 98 that implements multiplication in hardware (box ①), and one (box ②) that implements it
 99 using a trap handler. Note that since the configurability of this specification is first-class, i.e.
 100 expressed in Coq itself rather than in some configuration files of the build process, there is
 101 no code duplication between the two instantiations.

102 Parts of the exception handler (box ⑥) are implemented in the Bedrock2 source lan-
 103 guage [8] and compiled (⑦) using the Bedrock2 compiler, but the handler also needs some
 104 low-level operations that are not expressible in the Bedrock2 source language and are therefore
 105 implemented by-hand in assembly. That is, our handler (box ⑤) starts and ends in hand-
 106 written assembly, and calls a compiled Bedrock2 function in the middle. Our proof combines
 107 a program-logic proof about the Bedrock2 handler function, the compiler-correctness proof,
 108 and a proof about the assembly instructions, guaranteeing that all these parts have been
 109 put together correctly, and the final statement only mentions RISC-V semantics. All the
 110 other interfaces have been canceled out by combining the proofs and thus are not part of the
 111 trusted code base any more.

112 In addition to the two instantiations of the RISC-V semantics with and without hardware
 113 multiplication, our proof (but not the final statement) also uses a third instantiation (box ④)
 114 which does not have any CSRs (control and status registers, required by the exception
 115 mechanism). This third instantiation fails (with undefined behavior) on all CSR-related
 116 instructions. For the compiler, an axiomatization (box ③) of this instantiation was chosen

117 to simplify the proof, because the compiler does not emit any instructions that depend on
118 CSRs.

119 **3 The Toplevel Theorem Statement**

120 We can state the theorem (arrow ⑩ in Figure 1) as follows:

```
121 Theorem softmul_correct: forall (initialH initialL: MachineState) (post: State → Prop),
122   runsTo (mcomp_sat (run1 mdecode)) initialH post →
123   R initialH initialL →
124   runsTo (mcomp_sat (run1 idecode)) initialL (fun finalL ⇒
125     exists finalH, R finalH finalL ∧ post finalH).
126
127
```

128 It is phrased as a *specification preservation*¹ statement: If a machine with hardware
129 multiplication runs from an initial state `initialH` to states satisfying a postcondition `post`,
130 then every machine `initialL` with hardware multiplication, related to `initialH` by `R`, runs to
131 a low-level state `finalL` which, when translated back to a high-level state `finalH`, satisfies the
132 same postcondition.

133 The theorem uses `run1`, which defines how one single instruction is executed:

```
134 Definition run1(decoder: Z → Instruction): M unit :=
135   pc ← getPC;
136   inst ← Machine.loadWord Fetch pc;
137   Execute.execute (decoder (LittleEndian.combine 4 inst));;
138   endCycleNormal.
139
140
```

141 It is parameterized over the instruction decoder, which is instantiated with `mdecode`
142 (a decoder that supports the multiplication instruction) in the hypothesis and with `idecode`
143 (a decoder that returns `InvalidInstruction` for the multiplication instruction) in the conclusion
144 of the theorem.

145 The `mcomp_sat` function is of type `M unit → State → (State → Prop) → Prop` and asserts
146 that a monadic program (consisting of primitives used in `riscv-coq` such as `getRegister`,
147 `setRegister`, `loadByte`, etc), applied to some initial state, satisfies a postcondition, and `runsTo`
148 lifts it to an arbitrary (but finite) number of steps.² The predicate `R` (Figure 2) is used to
149 relate a high-level state (i.e. the state of a machine that supports multiplication in hardware)
150 to a low-level state (i.e. the state of a machine that implements multiplication in software
151 using a trap handler), and it also contains all the preconditions on how the low-level machine
152 needs to be configured. That is, `R` asserts that the two states have the same values for the
153 registers and the program counter, and that the memory (modeled as a partial map from
154 32-bit addresses to bytes) of the low-level machine contains everything of the high-level
155 memory, as well as the instructions of the exception handler and some scratch space that the
156 exception handler can use as its stack (which must be available even if the main program
157 has used up all of its stack). To define at which address in memory the handler and the
158 scratch space are located, RISC-V defines some control-and-status registers (CSRs) [2] that
159 our definition of `R` mentions:

- 160 ■ The CSR called `MTVecBase` is used to store the address of the trap handler (we use *direct*
161 mode where all exceptions set the PC to the same address, but RISC-V also has a *vectored*

¹ It can also be seen as a *small-step omnisemantics forward simulation* as defined in [6].

² `runsTo` is defined like the omnisemantics *eventually* operator [6].

```

Definition R(r1 r2: MachineState): Prop :=
  r1.(regs) = r2.(regs) ∧
  r1.(pc) = r2.(pc) ∧
  r1.(nextPc) = r2.(nextPc) ∧
  r1.(csrs) = map.empty ∧
  basic_CSRFields_supported r2 ∧
  regs_initialized r2.(regs) ∧
  exists mtvec_base scratch_end,
    map.get r2.(csrs) CSRField.MTVecBase = Some mtvec_base ∧
    map.get r2.(csrs) CSRField.MScratch = Some scratch_end ∧
    <{ * eq r1.(mem)
      * mem_available (word.of_Z (scratch_end - 256)) (word.of_Z scratch_end)
      * ptsto_bytes (word.of_Z (mtvec_base * 4)) softmul_binary }> r2.(mem).

```

■ **Figure 2** The predicate relating high-level states (multiplication implemented in hardware) to low-level states (multiplication implemented in software)

162 mode where the PC is set to the base address in this register plus an offset corresponding
 163 to the cause of the exception).
 164 ■ The CSR called MScratch is a read/write register dedicated for use by machine mode, and
 165 we use it to store the address of the *end* of the scratch space (we store the end address
 166 instead of the start address because it is used like a stack that grows downwards).

167 4 The Handler Code

168 The exception-handler code is implemented partially in handwritten assembly and partially
 169 in the Bedrock2 [8] source language and compiled to bytes by the Bedrock2 compiler. In
 170 order to *prove* the `softmul_correct` theorem, we use the correctness theorem of the Bedrock2
 171 compiler, but note that the *statement* of the `softmul_correct` theorem does not depend on the
 172 Bedrock2 language semantics or on anything related to the fact that we used the Bedrock2
 173 compiler, so the auditing burden for someone (who trusts the Coq proof checker) auditing
 174 our handler is much smaller, because one does not need to worry about the compiler, its
 175 language semantics, and its interaction with the assembly code.

176 The handwritten assembly of the handler is shown in Figure 3a. Since we want our
 177 software-emulated multiplication to behave as if it was implemented in hardware, we cannot
 178 make any assumptions about the remaining space on the user program’s stack, nor about
 179 whether the stack pointer `sp` contains any meaningful value at all. Therefore, we reserve
 180 a separate scratch space in memory just for our handler, and require that the control and
 181 status register (CSR) MScratch contains the address of that scratch space.

182 As its first action (in `handler_init`), the handler has to store all 32 registers of the user
 183 process by which it was triggered. It may only use registers that it has already saved, because
 184 otherwise it would destroy state of the user program. We therefore resort to tricks such
 185 as temporarily storing the user stack pointer in the MScratch CSR, and then temporarily
 186 storing it in the return address register. Such tricks are easy to get wrong (and we did, see
 187 section 8.2).

188 After `handler_init`, the registers 3 to 31 are saved to the scratch space as well, and then
 189 the Bedrock2-generated part is called by passing it the value of the CSR register `MTVa1`, which
 190 contains the invalid instruction that caused the exception, and a pointer to the scratch space
 191 in which we saved the registers.

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```
Definition handler_init := [[
  Csrw sp sp MScratch; (* swap sp and MScratch CSR *)
  Sw sp zero (-128);   (* save the 0 register (for uniformity) *)
  Sw sp ra (-124);    (* save ra *)
  Csrr ra MScratch;   (* use ra as a temporary register... *)
  Sw sp ra (-120);    (* ... to save the original sp *)
  Csrw sp MScratch;   (* restore the original value of MScratch *)
  Addi sp sp (-128)   (* remainder of code will be relative to updated sp *)
]].

Definition inc_mepc := [[
  Csrr t1 MEPC;
  Addi t1 t1 4;
  Csrw t1 MEPC
]].

Definition handler_final := [[
  Lw ra sp 4;
  Lw sp sp 8; (* Bug: used to be `Csrr sp MScratch`, which is wrong if Mul sets sp *)
  Mret
]].

Definition call_mul := [[
  Csrr a0 MTVal; (* argument 0: value of invalid instruction *)
  Addi a1 sp 0;  (* argument 1: pointer to memory with register values before trap *)
  Jal ra (Z.of_nat (1 + List.length inc_mepc + 29 + List.length handler_final) * 4)
]].

Definition asm_handler_insts := handler_init ++ save_regs3to31 ++
  call_mul ++ inc_mepc ++ restore_regs3to31 ++ handler_final.
```

(a) Assembly part of trap handler (embedded in Coq)

```
Definition softmul := func! (inst, a_regs) {
  a = a_regs + (inst>>15 & 31)<<2;
  b = a_regs + (inst>>20 & 31)<<2;
  d = a_regs + (inst>>07 & 31)<<2;
  unpack! c = rpmul(load(a), load(b));
  store(d, c)
}.

Definition rpmul := func! (x, e) ~> ret {
  ret = $0;
  while (e) {
    if (e & $1) { ret = ret + x };
    e = e >> $1;
    x = x + x
  }
}.

```

(b) Bedrock2 part of trap handler (using custom Coq notations to make it look similar to C)

■ **Figure 3** Trap handler code

```

Instance spec_of_softmul : spec_of "softmul" :=
  fnspec! "softmul" inst a_regs / rd rs1 rs2 regvals R,
  { requires t m :=
    mdecode (word.unsigned inst) = MInstruction (Mul rd rs1 rs2) ^
    List.length regvals = 32 ^
    seps [a_regs ↦ word_array regvals; R] m;
  ensures t' m' := t = t' ^
    seps [a_regs ↦ word_array (List.upd regvals (Z.to_nat rd) (word.mul
      (List.nth (Z.to_nat rs1) regvals default)
      (List.nth (Z.to_nat rs2) regvals default))); R] m' }.

```

■ **Figure 4** Specification of softmul function

192 The Bedrock2 code (Figure 3b) is written directly in Coq using the custom-notations
 193 feature, a C-like syntax, and operator precedence as suggested by whitespace. It extracts the
 194 three 5-bit fields of the instruction that indicate the two source registers (operands of the
 195 multiplication operation) and the destination register, respectively, and then calls another
 196 Bedrock2 function `rpmul` that implements multiplication in terms of addition, storing the
 197 result back into the scratch space. The `rpmul` function iterates over the bits of the second
 198 operand while repeatedly doubling the first operand, a technique sometimes called “Russian
 199 peasant multiplication.” Both `softmul` and `rpmul` are verified using the Bedrock2 program
 200 logic. The spec of the former is given in Figure 4.

201 Its pre- and postcondition are expressed in terms of an (unused) I/O trace `t` and the
 202 memory `m`, for which we assert a list of two separation-logic clauses (a word array corresponding
 203 to the scratch space containing the register values, and a generic frame `R` for the rest of the
 204 memory).

205 After the Bedrock2 part, the handwritten snippet `inc_mepc` runs. It increases the CSR
 206 called `MEPC`, which stores the address of the instruction that caused the exception, and upon
 207 returning from the trap handler (by the `Mret` instruction), execution will jump to `MEPC`, so we
 208 have to set it to one instruction (i.e., 4 bytes) past the multiplication instruction.

209 And finally, in `restore_regs3to31` and `handler_final`, the values of the user program’s
 210 registers are restored.

211 **5 Combining the Program-Logic Proofs and Compiler-Correctness** 212 **Proof**

213 By combining the program-logic proofs about the two Bedrock2 functions with the compiler-
 214 correctness theorem, we can prove that if we run the compiler within Coq to obtain a list
 215 of instructions `mul_insts`, these instructions satisfy the specification shown in Figure 5, a
 216 verbose but unsurprising specification, laying out calling-convention details.

217 Lines 5 to 6 specify in which registers the arguments need to be placed, and line 14
 218 requires that at address `a_regs`, there is an array of 32 words that store the values of the
 219 registers of the user program. Lines 18 to 20 state that after running `mul_insts`, the array at
 220 address `a_regs` storing the registers is updated at its `rd`’th index with the result of multiplying
 221 its `rs1`-th and `rs2`-th element, and line 23 states that the new registers of the processor (not
 222 the ones saved in memory) only differ from the original registers on the callee-saved registers.

223 Note that the conclusion on line 27 refers to the same machine as the conclusion of
 224 the top-level theorem in section 3, namely the one described by `(mcomp_sat (run1 idecode))`,

```

1 Lemma mul_correct: forall initial a_regs regvals invalidIInst R (post: State → Prop)
2   ret_addr stack_start stack_pastend rd rs1 rs2,
3   word.unsigned initial.(pc) mod 4 = 0 →
4   initial.(nextPc) = word.add initial.(pc) (word.of_Z 4) →
5   map.get initial.(regs) RegisterNames.a0 = Some invalidIInst →
6   map.get initial.(regs) RegisterNames.a1 = Some a_regs →
7   map.get initial.(regs) RegisterNames.ra = Some ret_addr →
8   map.get initial.(regs) RegisterNames.sp = Some stack_pastend →
9   word.unsigned ret_addr mod 4 = 0 →
10  word.unsigned (word.sub stack_pastend stack_start) mod 4 = 0 →
11  regs_initialized initial.(regs) →
12  mdecode (word.unsigned invalidIInst) = MInstruction (Mul rd rs1 rs2) →
13  128 ≤ word.unsigned (word.sub stack_pastend stack_start) →
14  seps [a_regs ↦ with_len 32 word_array regvals;
15        initial.(pc) ↦ program idecode mul_insts;
16        mem_available stack_start stack_pastend; R] initial.(MinimalCSRs.mem) ∧
17  (forall newMem newRegs,
18    seps [a_regs ↦ with_len 32 word_array (List.upd regvals (Z.to_nat rd) (word.mul
19          (List.nth (Z.to_nat rs1) regvals default)
20          (List.nth (Z.to_nat rs2) regvals default)));
21          initial.(pc) ↦ program idecode mul_insts;
22          mem_available stack_start stack_pastend; R] newMem →
23    map.only_differ initial.(regs) reg_class.caller_saved newRegs →
24    regs_initialized newRegs →
25    post { initial with pc := ret_addr; nextPc := word.add ret_addr (word.of_Z 4);
26          MinimalCSRs.mem := newMem; regs := newRegs }) →
27  runsTo (mcomp_sat (run1 idecode)) initial post.

```

■ **Figure 5** The correctness lemma of the compiler-generated part of the handler

225 or box ② in Figure 1. But to get there, two more proof steps (⑧ and ⑨) are needed:
 226 In order to keep the Bedrock2 compiler (somewhat) general, it was not proven against a
 227 specific instantiation of the riscv-coq semantics, but against an axiomatization (box ③) of
 228 the primitives used in riscv-coq such as `getRegister`, `setRegister`, `loadByte`, etc. However, to
 229 keep the Bedrock2 compiler proof manageable, the RISC-V machine state representation
 230 appearing in that axiomatization was hardcoded to a record type without CSRs (because
 231 compiler-emitted code never touches CSRs).

232 An additional problem requiring some proof effort to show compatibility is that the
 233 compiler correctness proof assumes a machine with hardware support for multiplication, but
 234 we want to run its code on one without. By inspecting the code that it generated, we can
 235 see that it did not output any multiplication instructions, but if it did, this would lead to a
 236 serious bug: If during the execution of the trap handler, a multiplication instruction was
 237 encountered, the trap handler would be recursively invoked again, infinitely many times.

238 We solve these two problems by introducing an intermediate helper machine (box ④)
 239 that uses the same state representation (without CSRs) as the compiler, and maintains an
 240 invariant `no_mul` saying that the memory region marked as executable (which only includes
 241 the compiled handler code in that instance) contains no multiplication instructions.

242 6 Correctness Proof of the Assembly Part

243 The assembly part of the handler is proven correct by induction over the `runsTo` hypothesis
 244 of `softmul_correct`. If the machine with hardware multiplication executes any instruction
 245 besides multiplication, we just need to show that after executing the same instruction on
 246 the machine with software multiplication, the `R` judgment is preserved, but we can do that
 247 once-and-for-all by inspecting each *primitive* of the riscv-coq spec (`getRegister`, `setRegister`,


```

Definition raiseExceptionWithInfo{A: Type}(isInterrupt exceptionCode info: t): M A :=
  pc ← getPC;
  (* hardcoded simplification: we only support machine mode and no interrupts *)
  addr ← getCSRField MTVecBase;
  setCSRField MTVal (regToZ_unsigned info);
  (* these two need to be set just so that Mret will succeed at restoring them *)
  setCSRField MPP (encodePrivMode Machine);
  setCSRField MPIE 0;;
  setCSRField MEPC (regToZ_unsigned pc);
  setCSRField MCauseCode (regToZ_unsigned exceptionCode);
  setPC (ZToReg (addr * 4));
  @endCycleEarly M t MM MW MP A.

```

■ **Figure 6** Specification (in riscv-coq) of what hardware does in case of an exception

248 loadByte, etc), instead of analyzing the much larger number of *instructions* that RISC-V
 249 has. The interesting case is when the machine with hardware multiplication encounters a
 250 multiplication instruction, and we have to show that the machine with software multiplication
 251 steps to a related state. We do so by first symbolically executing the specification of what
 252 the *hardware* does in case of an exception (Figure 6), which boils down to setting some CSR
 253 fields and then setting the PC to the exception-handler address found in the MTVecBase CSR.

254 After that, we symbolically execute the handwritten assembly instructions, using Coq’s
 255 proof context to keep track of all the facts that we know about the current state of the
 256 machine. For each assembly instruction, we encounter its specification in terms of the
 257 primitives of riscv-coq, and for each primitive, we have a helper lemma that updates our
 258 symbolic state. At the point where we reach the call to the Bedrock2-generated code, we
 259 apply the correctness lemma for the compiled trap handler. After that call, we step through
 260 more handwritten assembly instructions that restore the registers and then call the Mret
 261 instruction that jumps back to one instruction past the multiplication instruction that caused
 262 the exception. At that point, we need to prove that the symbolic state accumulated in the
 263 Coq proof context implies that the two machines are still related by R, which only works if
 264 there are no bugs in the handler code.

265 7 What If ...

266 To explain our specification from a different angle, we list a few potential bugs that an
 267 implementor could make, and show how they make our specification unprovable. Note that
 268 these are not bugs that actually occurred in our own implementation. For those, we refer to
 269 section 8.2. We present each potential bug as a question that begins with “What if ...

270 ■ ... the compiler used to compile the handler emitted a multiplication instruction, which
 271 would cause the handler to trigger itself recursively infinitely many times? When proving
 272 correctness of the handwritten assembly (section 6), when we get to the jump instruction
 273 that calls the code emitted by the Bedrock2 compiler, we need to apply the compiler
 274 correctness theorem (instantiated with the Bedrock2 part of our handler), but that
 275 theorem talks about execution on a machine with multiplication support, whereas the
 276 theorem we are about to prove is about execution on a machine without multiplication
 277 support. To make the proof work, we need to introduce box ④ and steps ⑧ and ⑨ in
 278 Figure 1 as explained in section 5, which at some point requires us to go through the
 279 concrete list of instructions emitted by the compiler and checking that none of them is a

280 multiplication instruction.

281 ■ ... the handler runs at a time when no stack exists or the stack does not have enough
 282 remaining space? The output of the Bedrock2 compiler contains a number that indicates
 283 the amount of stack space that the compiled code needs, and one hypothesis of the
 284 compiler correctness theorem is that at least that much space is available below the
 285 current stack pointer. In order to make sure this hypothesis holds, our trap handler uses
 286 a separate reserved scratch pad in memory as its stack, and when the correctness theorem
 287 for the handwritten assembly applies the instantiated compiler correctness theorem
 288 `mul_correct`, it has to prove that there are at least 128 bytes of space remaining in the
 289 scratch pad, as mandated by the hypothesis on line 13 in Figure 5.

290 ■ ... the assembly that calls compiled Bedrock2 code makes wrong assumptions about the
 291 calling conventions of the compiler, e.g. which registers are used to pass arguments, or
 292 whether they are passed on the stack, in which direction does the stack grow, or which
 293 registers are caller-saved? All these conventions are also captured in the intermediate
 294 lemma `mul_correct` in Figure 5.

295 ■ ... the handler forgot to increase MEPC, the CSR storing the address to which the
 296 machine jumps when we return from the exception handler, which would cause the
 297 faulting multiplication instruction to be run again, and would trigger the handler again?
 298 At the end of the handler correctness proof, this bug would lead to a mismatch between the
 299 state of the machine with multiplication support (whose program counter gets advanced
 300 past the multiplication instruction) and the state of the machine without multiplication
 301 support (whose program counter would still point to the multiplication instruction).

302 ■ ... we ran a user program using compressed instructions (2-byte instructions) on our
 303 system? The `riscv-coq` specification only supports the uncompressed instruction format,
 304 where all instructions are 4 bytes long. There is no single location where the spec explicitly
 305 says “compressed instructions are *not* supported” – it requires an attentive reader who
 306 notices that the whole spec never mentions compressed instructions. In this scenario, our
 307 trap handler would fail to decode the unsupported instruction, and arbitrary behavior
 308 would occur. If `riscv-coq` did support compressed instructions, and our handler correctly
 309 decoded them, that would still require it to correctly decide whether to increase the
 310 MEPC by 2 or 4, and like in the previous point, one would notice the mismatch during
 311 the proof.

312 **8 Evaluation**

313 We attempt to answer the following evaluation questions (and dedicate one subsection to
 314 each of them):

- 315 1. Does our verified trap handler run on a RISC-V system implemented by a third party?
- 316 2. Did our implementation contain bugs that our verification caught?
- 317 3. Did our implementation contain bugs that our verification failed to catch?
- 318 4. Was the effort required for verification lower than the effort for debugging would have
 319 been?

320 **8.1 Running Our Handler**

321 To validate that our verified handler actually runs on a system not implemented by ourselves,
 322 we first looked for small embedded RISC-V processors without multiplication support, but
 323 could not find any product with enough documentation in English to make us want to try it

324 out. Instead, we chose to test our code in the Spike RISC-V ISA simulator [3], which offers
 325 fine-grained control over which RISC-V extensions are enabled.

326 We want to test that our handler behaves as expected on a system that runs a simple
 327 C program with multiplications, compiled by a third-party compiler. We wrote a simple
 328 program which computes the factorial of a hardcoded number, and saves the result as well
 329 as a done flag to memory. We compiled it using the GNU RISC-V toolchain.

330 Our toplevel theorem applies to a list of bytes called `softmul_binary` (mentioned in Figure 2
 331 in the definition of the relation \mathbb{R}), representing a piece of position-independent RISC-V
 332 machine code. However, Spike expects as input an ELF file. We relied on the GNU RISC-V
 333 toolchain to transform our binary into an elf file, using a custom 25-line linker script.

334 For our theorem to be applicable, the conditions that the relation \mathbb{R} (Figure 2) imposes
 335 on `r2` (the machine without support for multiplication) must hold on our Spike machine.
 336 The first six conditions above the **exists** are related to the formalization and do not require
 337 any special setup action. The two lines below the **exists** require that the `MVecBase` and
 338 `MScratch` CSRs have suitable values, and we ensure this by running an assembly script at the
 339 beginning that initializes these two CSRs with addresses defined in our linker script. The
 340 last three lines are a bullet-point separation-logic clause list describing the memory, saying
 341 that it must contain all of the specification machine’s memory `r1.mem`, as well as 256 bytes
 342 of scratch memory at the address in the `MScratch` CSR and the `softmul_binary` at the address
 343 in the `MVecBase` CSR. Our linker script, together with the memory-layout command-line
 344 argument we pass to Spike, ensures that these conditions hold.

345 Spike comes with its own small language of debugger commands, and we used it to run
 346 the system until the done flag in memory is 1, then print the value of the memory at the
 347 address where we expect the result, and also print the value of the CSR register `minstret`,
 348 the number of retired instructions, to see how many instructions were executed.

349 No matter whether we invoked Spike with or without multiplication enabled, we observed
 350 the same result for `factorial(5)`, namely 120. With multiplication enabled, the number of
 351 instructions was 87, and with multiplication disabled, the number of instructions increased
 352 to 787, which shows that our handler indeed ran. As an additional sanity check, we also
 353 confirmed that it stops working if we set the `MVecBase` CSR to a different value.

354 Therefore, at least for this one simple example, we can answer question 1 with ‘yes’.

355 8.2 Bugs Caught During Verification

356 At the end of the proof that steps through the handwritten handler assembly, we need to
 357 prove that the symbolic state accumulated in the Coq proof context implies that the two
 358 machines are still related by \mathbb{R} , which only works if there are no bugs in the handler code (see
 359 end of section 6). At that point, we found two interesting bugs. The first one was that we
 360 forgot to reset the `MScratch` CSR, so one invocation of the exception handler works fine, but
 361 the next one will use a wrong address for its scratch space. The second bug was the corner
 362 case where the multiplication instruction stores its result into the stack pointer. In that case,
 363 we must not override the stack pointer with the original stack pointer that we swapped into
 364 the `MScratch` register at the beginning of the handler.

365 We also found two more obvious bugs related to when to set the stack pointer and what
 366 stack-pointer offsets to use.

367 So we can answer question 2 with ‘yes’.

368 8.3 Bugs Encountered While Trying to Run It

369 We split the development of our experiment into two phases: First, we set up the linker script,
 370 with the trap handler already in place, but inactive, because we enabled the M extension.
 371 Once this experiment produced the expected output, we deactivated the M extension, so
 372 that our handler would run.

373 Getting phase 1 to work required some debugging. The most difficult part was to
 374 understand how to pass the linker-script defined address of the heap memory to the C
 375 program, and required reading the relevant page³ of the GNU Linker’s manual, which starts
 376 by saying that “accessing a linker script defined variable from source code is not intuitive”,
 377 and further down explains that “when you are using a linker script defined symbol in source
 378 code you should always take the address of the symbol, and never attempt to use its value”.

379 None of the code involved in phase 1 was verified, so it is not surprising that debugging
 380 was required. And to our delight, in phase 2, as soon as we disabled the M extension, our
 381 verified trap handler worked on the first try, and no debugging was needed at all.

382 So, to answer question 3, there were bugs in the unverified part, but no bugs in the
 383 verified part.

384 In the future, it would be interesting to also verify ELF file generation, and we believe
 385 that this could have prevented the above bug.

386 8.4 Effort

387 For lack of better measures, we resort to lines of code counts as a very approximate measure
 388 of effort. Table 1 lists the lines of code counts of the different components.

389 It suggests that to produce 76 lines of verified code, a total of 3331 lines of code was
 390 necessary, which is more than a 40× blowup. This ratio looks not very appealing, but it still
 391 seems fair to say that for tricky code, large proofs are sometimes needed. We also have some
 392 (potentially alleviating) remarks for each row of the table:

- 393 ■ The RISC-V helper instance is not referenced by the toplevel theorem statement, but
 394 acts as a bridge between the RISC-V spec used by the Bedrock2 compiler (whose state
 395 does not contain any CSRs) and the one used in the toplevel theorem (whose state does
 396 have CSRs). Additionally, the helper instance maintains the invariant that no executable
 397 instructions are from the M extension, which is important during the execution of the
 398 trap handler, because if the trap handler contained a multiplication instruction, the trap
 399 handler would be invoked recursively over and over again. The helper instance and its
 400 accompanying lemmas are mostly copied from the one used in the compiler, and careful
 401 refactoring to share the code with the compiler could considerably reduce this count,
 402 which also means that these lines were low-effort to produce.
- 403 ■ To verify multiplication and a simple instruction decoder in Bedrock2, we used the original
 404 Bedrock2 program logic [8], which only automates the application of weakest-precondition
 405 rules, but does not provide any automation for sidecondition solving. Using a framework
 406 that provides more automation would have reduced this proof size.
- 407 ■ A large chunk of the proof lines (1454) is in the correctness proof of the trap handler
 408 parts written in assembly. The reason for this verbosity might be that, to our knowledge,
 409 this project is the first within the Bedrock2 ecosystem to verify more than two or three
 410 lines of assembly at a time, so there was no assembly-specific framework available. About

³ <https://sourceware.org/binutils/docs/ld/Source-Code-Reference.html>

	impl	spec	proof	total
RISC-V helper instance	0	101	309	410
Multiplication in Bedrock2	8	5	83	96
Instruction decoder in Bedrock2	7	27	80	114
Trap handler in assembly	36	28	1454	1518
Compiler compat & invocation	14	47	716	777
Toplevel theorem	11	18	147	176
Excluded (imports & comments)				240
Total	76	226	2789	3331

■ **Table 1** Lines of code counts, excluding the dependencies (coqutil, riscv-coq, Bedrock2, and the Bedrock2 compiler)

411 two thirds of the proof code could probably be factored out into a framework that would
 412 be reusable for other assembly programs as well. We also did not spend too much
 413 time on sidecondition automation, which could further reduce the number of proof lines.
 414 We conjecture that in a more mature assembly verification framework, the assembly
 415 part trap handler proof might be as short as maybe 100 lines of code. Moreover, the
 416 code to proof ratio also looks bad because we count the number of lines of Coq code
 417 rather than the number of assembly instructions, which matters for `save_regs3to31` and
 418 `restore_regs3to31`: Each of these is just a two-line functional program, but expands to 29
 419 assembly instructions.

- 420 ■ The compiler compat & invocation code deals with the different RISC-V instances and
 421 decoders, and also applies the Bedrock2 compiler’s correctness theorem for the instruction
 422 decoder and multiplier implemented in Bedrock2. It consists of important but not
 423 particularly interesting bookkeeping that quickly adds up to many lines of proof.
- 424 ■ Finally, the toplevel theorem puts everything together. It requires some helper lemmas
 425 that could probably be generalized and moved to a library, but the fact that these lemmas
 426 were not already present in any library used in the Bedrock2 ecosystem seems fairly
 427 representative of the general verification experience, so it seems fair to count these lines.

428 Finding the bugs described in section 8.2 through debugging (especially the first two)
 429 might have been quite hard, but would probably still not have taken as long as our verification
 430 effort took, so the answer for question 4 is probably a ‘no’.

431 But we can imagine a promising world where the proof burden becomes lower than the
 432 debugging burden and verification becomes a part of most systems developer’s toolboxes.

433 9 Related Work

434 A number of projects have attempted to verify the interaction between (some or all of)
 435 C code, its compilation, handwritten assembly code, and trap handlers.

436 In the context of the Verisoft project, Alkassar et al. [4] verified a virtual memory system
 437 that can swap out virtual memory pages onto disk. If an address is accessed that currently
 438 is on disk, a page fault is triggered, and a verified page fault handler runs. Their correctness
 439 statement says that a physical machine with the page fault handler can simulate a virtual
 440 machine (by which they mean a machine that provides to a user process a linear memory
 441 covering the whole address space). Their handler is implemented in C0 (a subset of C)
 442 with some inline assembly, which is modeled as external calls that modify additional state

443 that cannot be modified directly from C0. That is, they call assembly from C, whereas we
444 chose the opposite direction, calling C (or the C-like language Bedrock2, in our case) from
445 assembly. In their project, saving and restoring of registers before and after the handler are
446 not implemented in assembly and verified like we do, but are part of the semantics of the
447 physical machine.

448 BabyVMM [17] proves correctness of a simple virtual memory manager by showing that
449 for all kernel implementations, linking the kernel with the virtual memory manager and
450 running it on a machine with only physical memory (“hardware model” HW) behaves like
451 running the kernel on a machine with an address space whose lower part is physical memory,
452 and whose upper part is virtual memory (“address space model” AS). It is implemented in a
453 C-like language, and no compiler nor assembly code appears in the formalization. Instead,
454 the theorem is stated in terms of C semantics. It also does not mention any page fault
455 handlers.

456 The verified microkernel seL4 [12] is implemented in C, but some small parts are hand-
457 written assembly and are not verified [14, sections 4.4 and 4.8]. Contrary to our approach of
458 using a verified compiler, they apply translation validation to the binary generated by GCC
459 and certify using SMT solvers that it behaves like the C program.

460 CertiKOS [10, 11, 7] is a verified OS kernel. By means of certified abstraction layers, it
461 fully captures the behavior of each component in a deep specification, so that from the outside,
462 it does not matter whether the component is implemented in C or in assembly, thus achieving
463 interoperability at the proof level between C and assembly. Its correctness is expressed as
464 a contextual refinement, based on CompCert’s [15] notion of a backward simulation, but
465 extended with a universal quantification over all possible surrounding programs (contexts):
466 It states that for all assembly programs, all behaviors of that assembly program when
467 linked with the low-level kernel can be simulated by the same program when linked with
468 the high-level kernel specification. It relies on a notion of linking, and uses CompCert’s
469 formalization of assembly, which is still fairly high-level compared to binary machine code,
470 eg. jumps use labels instead of offsets or addresses, and there are instructions that allocate
471 and free a stack frame that do not correspond to any machine instructions. CompCert’s
472 assembly (which is used to model CertiKOS’ lowest layer) also does not model CSRs, whereas
473 riscv-coq, on which our project is based, does, so to model trap handlers at our level of detail,
474 the assembly (or machine) model would have to be extended.

475 CompCertELF [18], a different project by the same group, extends CompCert to also
476 cover machine code generation and uses a more realistic memory model, without the stack
477 frame allocation/freeing instructions mentioned above. As far as we know, CompCertELF
478 has not (yet) been integrated with CertiKOS, and is not publicly available. If it was, and if
479 we managed to make CompCertELF compatible with our project, it could have helped to
480 prevent the bug (section 8.3) we encountered in our unverified usage of the GNU linker to
481 turn our plain binary into an ELF file.

482 Goel et al. [9] verify a subset of the instructions of an x86 processor which decodes x86
483 instructions and translates them into micro-operations before executing them. For the more
484 complex instructions, the generated micro-operations contain a trap that causes a jump to
485 microcode stored in a ROM. Similarly to our theorem, they prove that this processor behaves
486 as if there were no micro-operations, traps or microcode, and instructions were executed
487 according to a high-level x86 specification.

488 In the CakeML world, there is ongoing work on Pancake [16], a low-level language for
489 device drivers. It comes with a verified compiler that reuses significant parts of the backend
490 of the CakeML compiler [13]. It does not support inline assembly, and the paper does not

491 report on verified handwritten assembly code that would call Pancake-generated assembly.
 492 Instead, all interaction with devices is modeled as external calls at the moment.

493 **10 Conclusion and Future Work**

494 We have shown a pleasantly simple way of specifying the correctness of a trap handler
 495 that emulates unsupported instructions in software, and proved that our implementation of
 496 such a trap handler combining handwritten assembly and compiler-generated code satisfies
 497 this specification by combining symbolic-evaluation proofs about assembly and Bedrock2
 498 programs with the correctness proof of the Bedrock2 compiler, and by proving that the
 499 output of the Bedrock2 compiler, which assumes a machine without CSRs and with hardware
 500 support for multiplication, also runs correctly on a machine with CSRs, but without hardware
 501 support for multiplication.

502 Being able to state and prove a theorem saying that user programs on a machine without
 503 hardware support for multiplication but a trap handler to emulate it behave as if they ran
 504 on a machine with hardware support for multiplication constitutes a first step towards the
 505 more ambitious goal of thoroughly proving correctness of a virtual memory system, stated
 506 in a similar flavor by saying that user programs running on a system with virtual memory
 507 (implemented by a combination of hardware, assembly, and C) behave as if they were running
 508 on a machine where the user program can use the full physical address space.

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